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Econometrics

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Review

Endogeneity

Instrumental Variable

IV Estimator

2 SLS

Multiple Endogenous Variables

Weak Instrument

Over-identifying Restriction

Cases of M.E.

Case 1:  $y = y^* + e_0 \implies \text{Var} = \sigma_u^2 + \sigma_{e_0}^2$  } Loss of efficiency  
But unbiased

Case 2:  $x = x^* + e_1, \text{Cov}(x, e_1) = 0$

Case 3:  $x = x^* + e_1, \text{Cov}(x^*, e_1) = 0$

Case 2  $\implies y = \beta_0 + \beta_1 x + (u - \beta_1 e_1)$

$$\implies \text{Var} = \sigma_u^2 + \beta_1^2 \sigma_{e_1}^2$$

Case 3  $\implies \text{Cov}(x^*, e_1) = 0$

$$\text{Cov}(x - e_1, e_1) = 0 \implies \text{Cov}(x, e_1) = \text{Cov}(e_1, e_1) = \sigma_{e_1}^2$$

$\neq 0$

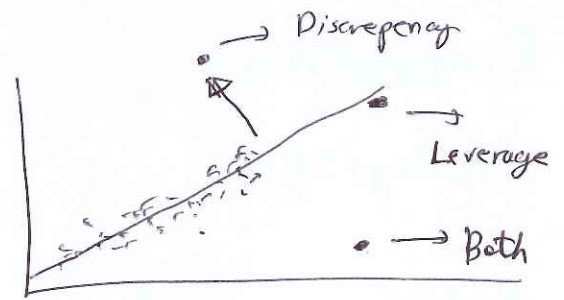
$$\implies E(\hat{\beta}_1) = \beta_1 \frac{\sigma_{x^*}^2}{\underbrace{\sigma_{x^*}^2 + \sigma_{e_1}^2}_{< 1}} \quad (\text{Attenuation Bias})$$

① Missing observations are a problem if they are a function of the Y variable

②

## Outliers

$$\text{Influence} = \text{Leverage} \times \text{Discrepancy}$$



To detect: `rvfplot`

(STATA) `Lvr2plot` (plots lev against dis)

→ `reg`

→ `predict resid, rstudent`

Use `qreg y x1 x2` (based on median)

`rreg y x1 x2` (assigns less weights to outliers)

## Endogeneity

X is endogenous if

$$\text{Cov}(X, u) \neq 0$$

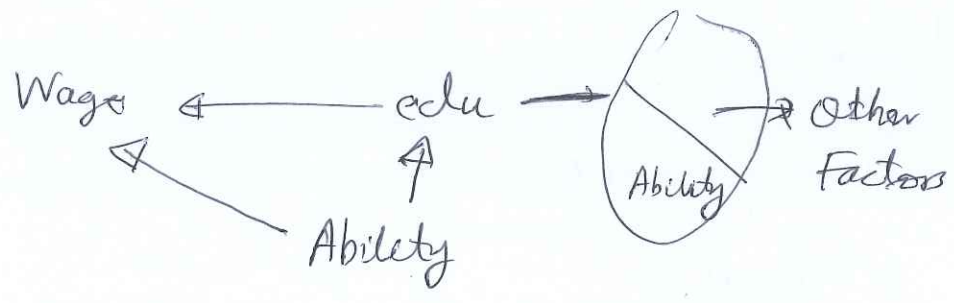
Wage studies have a problem of endogeneity due to the fact that ability is unobservable.

In case of price-quantity relationship



price and quantity both affect each other.

Hence quantity is affected by price and other factors such as ~~what~~ weather or political tensions.



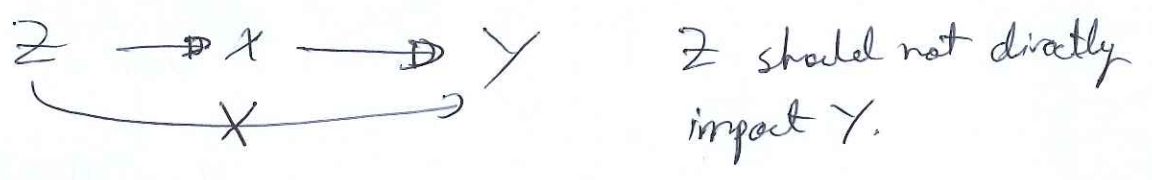
What Problem of Endogeneity

- 1. OVB  $E[\hat{\beta}_1] = \beta_1 + \beta_2 \delta$
- ~~2. Missing Data~~
- 2. Non-random Missing Data / Self-Selection
- 3. Measurement Error
- 4. Functional Form Misspecification

$$y = \beta_0 + \beta_1 x + u$$

Qualities of an Instrument

- 1.  $Cov(Z, X) \neq 0$  (Relevance)
- 2.  $Cov(Z, u) = 0$  (Exogeneity)



Z should not directly impact Y.

If Z affects Y then  $Cov(Z, u) \neq 0$   
(as u is a part of Y).

$$Cov(y, z) = \beta_1 Cov(x, z) + Cov(y, z)$$

$$\Rightarrow \hat{\beta}_1^{IV} = \frac{Cov(y, z)}{Cov(x, z)} \sim \beta_1 = \frac{Cov(y, x)}{Cov(x, x)} \stackrel{=0}{\text{cancel}}$$

# 2 stage Least Squares

1.  $x = \gamma_0 + \gamma_1 z + e$

$x = \hat{x} + e$   
↳ variation due to  $z$

2.  $y = \beta_0 + \beta_1 \hat{x} + u$

$\gamma_1 = \frac{\text{Cov}(x, z)}{\text{Var}(z)}$

$$\begin{aligned} \beta_1 &= \frac{\text{Cov}(y, \hat{x})}{\text{Var}(\hat{x})} = \frac{\text{Cov}(y, \gamma_0 + \gamma_1 z)}{\text{Var}(\gamma_0 + \gamma_1 z)} \\ &= \frac{\gamma_0 \text{Cov}(y, 1) + \gamma_1 \text{Cov}(y, z)}{\gamma_0^2 \text{Var}(z)} \\ &= \frac{\text{Cov}(y, z)}{\gamma_0 \text{Var}(z)} \\ &= \frac{\text{Cov}(y, z)}{\text{Var}(z)} \times \frac{\text{Var}(z)}{\text{Cov}(x, z)} \\ &= \frac{\text{Cov}(y, z)}{\text{Cov}(x, z)} \end{aligned}$$

# Multiple Endogenous Variables

If we have two endogenous variables then we will need two instrumental variables.

If # endogenous  $\geq$  # instruments

$\Rightarrow$  under-identification

$<$   $\Rightarrow$  over identification

$=$   $\Rightarrow$  exact identification

If we have more instruments then we can run different IV regressions and test our results from different regressions if they confirm to each other.

## Weak Instrument

$$\beta^{IV} = \frac{\text{Cor}(y, z)}{\text{Cor}(x, z)}$$

$\rightarrow$  small

to check for weak instruments run regression of  $x$  on  $z$ 's and check for the  $f$ -statistic.

