NYUAD Mathematics Club

13 February 2016

Problem 1. Let x, y, z be positive integers. Prove that it is impossible to have all of the three numbers $x^2 + y + z$, $y^2 + x + z$, $z^2 + y + x$ to be perfect squares.

Problem 2. Let ABC be an acute triangle with altitudes AD, BE, CF, and let O be the center of its circumcircle. Show that the segments OA, OF, OB, OD, OC and OE dissect the triangle ABC into three pairs of triangles that have equal areas.

Problem 3. For a positive integer m denote by $S(m)$ and $P(m)$ the sum and product, respectively, of the digits of m. Show that for each positive integer n, there exist positive integers a_1, a_2,..., a_n satisfying the following conditions:

$$S(a_1) < S(a_2) < ... < S(a_n)$$

and

$$S(a_i) = P(a_{i+1}) \text{ for } (i = 1, 2, ..., n).$$

(We let $a_{n+1} = a_1$.)

Notes:

If you can, you are particularly encouraged to submit your solutions in advance to okm212@nyu.edu