(3)

Let k be sufficiently large $2^k > 2(k+n-1)$

for $i = 2, 3, \dots n$,

a_i is such that 2 appears $(k+i-2)$ times

1 appears $2^{k+i-1} - 2(k+i-2)$

Then $S(a_i) = 2^{k+i-1}$

$P(a_i) = 2^{k+i-2}$

Define a_1 such that 2 appears $k+n-1$ times

1 appears $2^k - 2(k+n-1)$ times

Then $S(a_1) = 2^k$

$P(a_1) = 2^{k+n-1}$

(1) $(x+x_1)^2 = x^2+y+z \implies y+z = 2xx_1 + x_1^2$

$x_1^2 = y+z-2xx_1$

Similarly $y_1^2 = x+z-2yy_1$

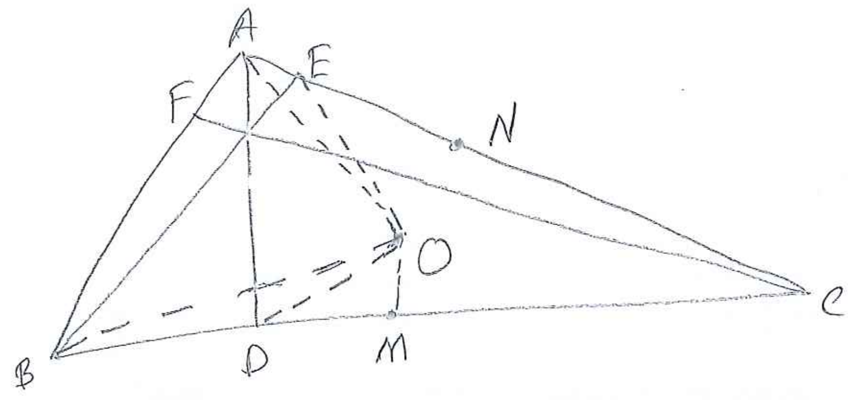
$z_1^2 = x+y-2zz_1$

Since $x_1, y_1, z_1 \geq 1 \implies x_1^2+y_1^2+z_1^2 \leq 2(x+y+z) - 2x - 2y - 2z$

$= 0$

Hence $x_1 = y_1 = z_1 = 0$ (A contradiction)

(2)



(i) Let M, N be midpoints of BC, AC

$\Rightarrow M\hat{O}C = \frac{1}{2} B\hat{O}C = E\hat{A}B$

$\Rightarrow O\hat{M}C = 90^\circ = A\hat{E}B$

$\Rightarrow \Delta OMC \approx \Delta AEB$

$\Rightarrow \frac{OM}{AE} = \frac{OC}{AB}$

Hence $\Delta ONA \approx \Delta BDA$

$\Rightarrow \frac{ON}{BD} = \frac{OA}{BA}$

Sine $OA = OC \Rightarrow \frac{ON}{BD} = \frac{OA}{BA} = \frac{OC}{AB} = \frac{OM}{AE}$

$\Rightarrow BD \cdot OM = AE \cdot ON$

$\Rightarrow$ Observe that $\Delta(OBD) = \frac{1}{2}(BD)(OM) = \frac{1}{2}(AE)(ON) = \Delta(OAE)$

$\therefore$ Similarly, the solution works for other two pairs.