Problem 2

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Problem 2 (1.2): Prove the the following propositions: Two states i and j of a Markov Chain *communicate* if and only if $L_{ij} > 0$ and $L_{ji} > 0$.

Recall that, states i and j *communicate* if $\exists n, m$ such that $p_{ij}^{(n)} > 0$ and $p_{ji}^{(m)} > 0$ **(Def. on page 14)**.
$L_{ij} > 0$ implies that $\sum_{n=1}^{\infty} K_{ij}^{(n)} > 0$ by **(1.15) pg. 15**.

We state the following for a particular $n^{'} \leq n$:

$$p_{ij}^{(n^{'})} = \sum_{v=1}^{n^{'}} K_{ij}^{(v)} p_{jj}^{(n^{'}-v)} \tag{1}$$

$$\geq K_{ij}^{(n^{'})} p_{jj}^{(0)} = K_{ij}^{(n^{'})} \tag{2}$$

Since we know that $p_{ij}^{(n)} > 0$ and that $n^{'} \leq n$ we can say that $K_{ij}^{(n^{'})} > 0$. Hence we can be certain that, $L_{ij} > 0$. By symmetry we can prove that $L_{ji} > 0$.