NYUAD Mathematics Club

20 February 2016

Problem 1. The real numbers $x_1, x_2, ..., x_{2016}$ satisfy:

$$x_1 + x_2 = 2a_1,\ x_2 + x_3 = 2a_2,\ ...,\ x_{2016} + x_1 = 2a_{2016}$$

where $a_1, a_2, ..., a_{2016}$ is a permutation of $x_1, x_2, ..., x_{2016}$. Prove that $x_1 = x_2 = ... = x_{2016}$.

Problem 2. In Abu Dhabi there are three schools called A, B and C, each of which is attended by at least one student. Among any three students, one from A, one from B and one from C, there are two knowing each other and two not knowing each other. Prove that a student from A, B **or** C knows knows all students from B, C **or** A, **respectively**.

Problem 3. Let $f : R \to R$ be a function such that

$$f(f(x)) = x^2 - x + 1$$

for all real numbers x. Determine $f(0)$.

Problem 4. Let $a_1, a_2, ..., a_n$, be non-negative reals such that $\sum_{i=1}^{n} a_i = n$. Prove the inequality:

$$\textstyle\sum_{i=1}^{n} \frac{a_i}{a_i^3+8} \leq \frac{n}{9}.$$

Notes: If you can, you are particularly encouraged to submit your solutions in advance to okm212@nyu.edu