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logit of educ exper

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = \frac{\beta_0 + \beta_1 x}{z}$$

$$\log\left(\frac{\hat{p}}{1-\hat{p}}\right) = z \Rightarrow p = \frac{e^z}{1+e^z}$$

if educ = 5, you will put  $x=5$  to find  $z$   
and then use  $z$  to find  $\hat{p}$ .

STATA: margins, at(educ = (1(2)30))

to graph: marginsplot

↑ point  
→ increment  
→ end

## Heteroskedasticity (Het.)

- What is Het?
- Consequences?
- Detection
  - visual
  - formal test
    - Breusch-Pagan
    - White
- Solutions
  - GLS/WLS
  - FGLS
  - Robust S.E.

## L.M. Test

(Lagrange Multiplier Test)

$$1. y = \beta_0 + \beta_1 x_1 + \hat{u}$$

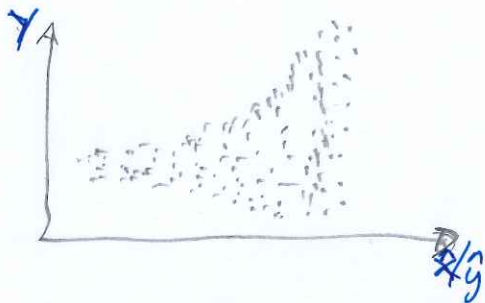
$$2. \hat{u} = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + v_i$$

$$3. N \cdot R_{\hat{u}}^2 \sim \chi^2 \text{ [Chi-Squared Test]}$$

$$A4 \Rightarrow \text{Var}(u_i | x_i) = \sigma^2$$

$$\text{Var}(\hat{\beta}_i) = \frac{\sigma^2}{SST_X}$$

$$\text{if } A4 \Rightarrow V(v_i | x_i) = \sigma_i^2$$



$$(t\text{-stat})^2 = F\text{-stat}$$

Het.  $\Rightarrow$  OLS is not BLUE

STATA Use `rvfplot` to observe residuals.

$$\text{Var}(\hat{u}) = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \dots$$

$$H_0: \delta_1 = 0, \delta_2 = 0$$

$$\sigma_i^2 = E[u_i^2]$$

$\hat{u}^2$  as a proxy for variance

$$\hat{u}^2 = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + v_i$$

1. reg  $y$   $x_1$   $x_2$
2.  $\hat{u}$
3.  $\hat{u}^2$
3. ~~reg~~ reg  $\hat{u}^2$   $x_1$   $x_2$
4. test  $\delta_1 = \delta_2 = 0$

(F-test or L.M. test)

$$\text{Var}(u) = \sigma^2$$

$$\text{Var}(\hat{u}) = \frac{\sigma^2}{n_j}$$

We might feel that we have OVB if we cannot ~~observe~~ <sup>detect</sup> het.

⊙ All other assumptions hold for het. (apart from homos. assumption).

## STATA

reg y educ exper

predict uhat, res

gen uhat2 = uhat<sup>2</sup>

reg uhat<sup>2</sup> educ exper

$$\frac{X' u}{N} = \frac{R_u^2}{F_{stat}}$$

STATA: hettest (after ~~reg~~ wage educ exper)

Problem with Breusch-Pagan Test

⇒ Only looks at the linear relation

## White Test

$$\hat{u}^2 = \delta_0 + \delta_1 x_1 + \delta_2 x_2 + \delta_3 x_1^2 + \delta_4 x_2^2 + \delta_5 x_1 x_2 + \epsilon$$

1. reg y x1 x2

2.  $\hat{u} \rightarrow \hat{u}^2$

3. reg  $\hat{u}^2$  x1 x2 x1<sup>2</sup> x2<sup>2</sup> x1x2

4. F-test or L.M.

Problem: ~~Use~~ You loose degrees of freedom

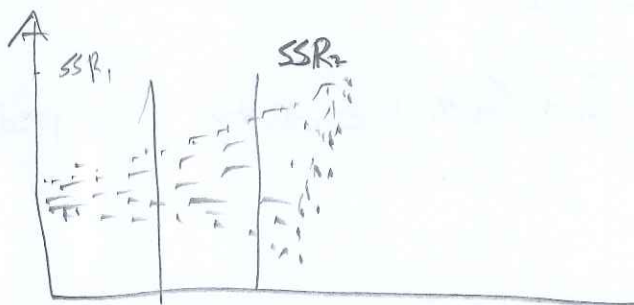
## Park Test

Uses log-log specification.

## Special Case of White Test

$$3. \text{reg } \hat{u}^2 \text{ } \hat{y} \text{ } \hat{y}^2$$

## Goldfeld-Quandt Test



$$\text{Test for } \Rightarrow \frac{SSR_1}{SSR_2} \sim 1$$

## Solutions

$$\text{Var}(u|x) = \underbrace{x_1 \sigma^2}_{\sigma^2}$$

Weighted-Least Squares

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

$$\frac{y}{\sqrt{x_1}} = \frac{\beta_0}{\sqrt{x_1}} + \frac{\beta_1 x_1}{\sqrt{x_1}} + \frac{\beta_2 x_2}{\sqrt{x_1}} + \frac{u}{\sqrt{x_1}} \quad (\text{Divide by } \sqrt{x_1})$$

$$E\left[\left(\frac{u}{\sqrt{x_1}}\right)^2\right] = \frac{x_1 \sigma^2}{x_1} = \sigma^2$$

## Feasible Generalized Least Squares

Suppose: 
$$\text{Var} = \sigma^2 e^{\underbrace{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon}_{h_i}}$$

1. reg  $y$   $x_1$   $x_2$

2.  $\hat{u} \rightarrow \hat{u}^2 \rightarrow \ln(\hat{u}^2)$

3. reg  $\ln(\hat{u}^2)$   $x_1$   $x_2$

4.  $\hookrightarrow \hat{g} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$  [predict  $\hat{g}$ ,  $\epsilon$ ]

5.  $\hat{h} = e^{\hat{g}}$

6. reg  $\frac{y}{\sqrt{h_i}}$   $\frac{x_1}{\sqrt{h_i}}$   $\frac{x_2}{\sqrt{h_i}}$