Econometrics (23/Feb/2016)

→ Hypothesis Testing

$H_0: \mu = 30$

$\bar{X} = 35$

$Var = 25 \Rightarrow SD = 5$

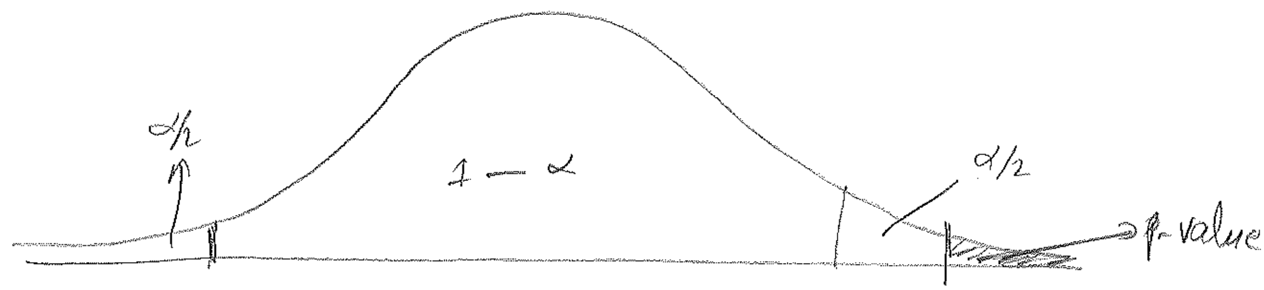
$n = 10$

$Var(\bar{X}) = \dfrac{25}{10} = 2.5$

$$T = \Phi\left(\frac{35-30}{\sqrt{\frac{\sigma^2}{n}}}\right) = \Phi\left(\frac{5}{\sqrt{2.5}}\right) = \Phi(3.16)$$

$$\approx 0.06\%$$

∴ As number of observation increases in a t-statistic, the p-value converges to the z-score.

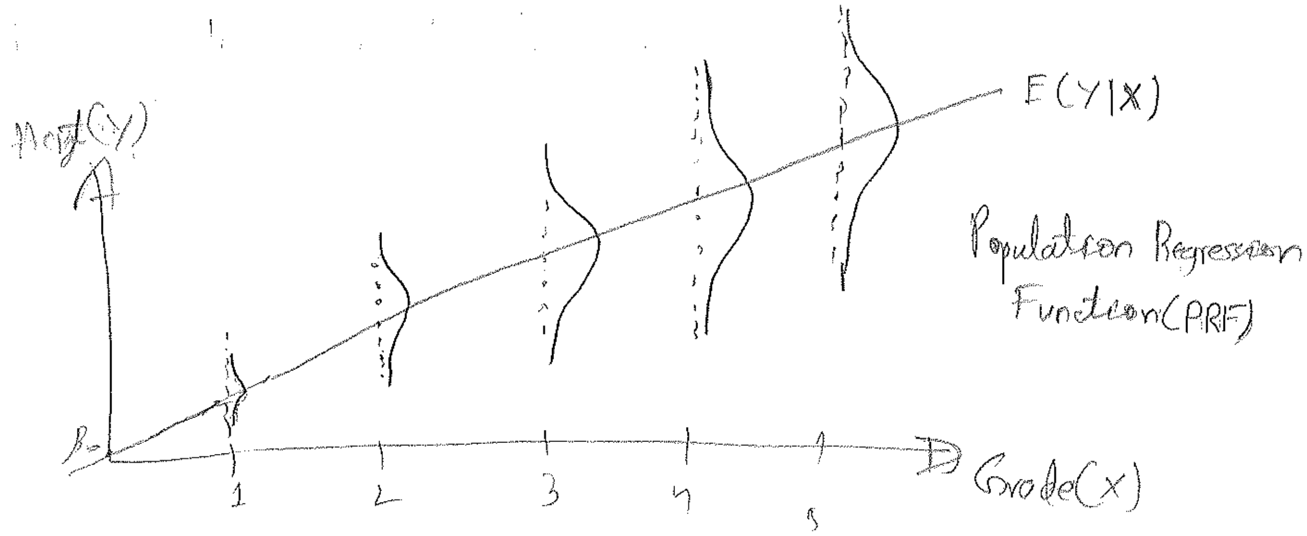


If $p < \frac{\alpha}{2}$, we reject the null hypothesis.

$p = 0.10 \Longrightarrow$ Some evidence that H_0 is false

$p = 0.05 \Longrightarrow$ strong evidence that H_0 is false

$p = 0.01 \Longrightarrow$ Very strong evidence that H_0 is false.



(It is also reasonable to assume that SD will be higher at bigger grades).

$$Y = \beta_0 + \beta_1 X + u$$

- β_0 → Constant/Intercept
- β_1 → Slope/Parameter
- u → Error Term (Stochastic Variable) (Unobserved)
- Y → Dependent Variable (Endogenous) (Explained Variable)
- X → Independent Variable (Exogenous), (Explanatory Variable)

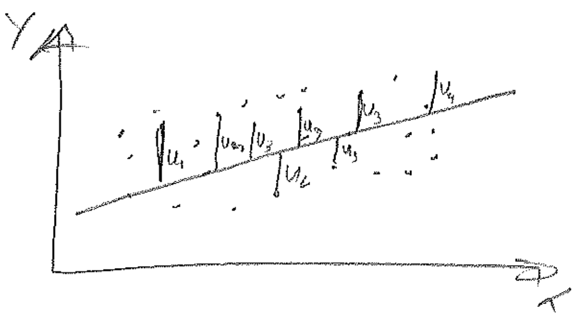
PRF $\Longrightarrow$ $E[Y|X] = \beta_0 + \beta_1 X$ (Since $E[u] = 0$ is an assumption)

Derivation - OLS

$$\min Q = \sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2$$

$$Y = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{u}$$

We select β_0, β_1 that minimises Q.



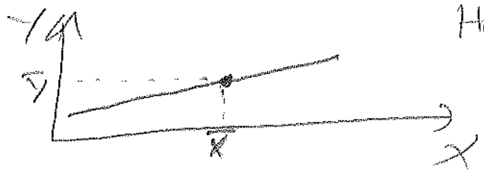
$$\frac{dQ}{d\beta_0} = (-2)\sum(Y - \hat{\beta}_0 - \hat{\beta}_1 X) = 0$$

$$\sum(Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i) = 0$$

$$\left(\sum Y\right) - n\beta_0 - \hat{\beta}_1\left(\sum X\right) = 0$$

$$\Longrightarrow \boxed{\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}}$$

Here



Hence it passes through $\overline{Y}$ and $\overline{X}$.

$$\frac{dQ}{d\beta_1} = -2\sum(Y_i - \overline{\beta}_0 - \overline{\beta}_1 X_i)(X_i) = 0$$

$$\sum(Y_i X_i - \hat{\beta}_0 X_i - \hat{\beta}_1 X_i^2) = 0$$

$$\left(\sum Y_i X_i\right) - (\overline{Y} - \beta_1 \overline{X})\left(\sum X_i\right) - \beta_1\left(\sum X_i^2\right) = 0$$

$$\left(\sum Y_i X_i\right) - \overline{Y}\left(\sum X_i\right) + \beta_1 \overline{X}\left(\sum X_i\right) - \beta_1\left(\sum X_i^2\right) = 0$$

$$\sum\left[(Y_i - \overline{Y})(X_i)\right] + \beta_1 \sum\left[(\overline{X} - X_i)(X_i)\right] = 0$$

$$\Longrightarrow \beta_1 = \frac{\sum(Y_i - \overline{Y})(X_i)}{\sum(X_i - \overline{X})(X_i)}$$

Since $\sum(Y_i - Y)(X_i) = \sum(Y_i - \overline{Y})(X_i - \overline{X})$

and $\sum(X_i - \overline{X})(X_i) = \sum(X_i - \overline{X})^2$

Hence $$\beta_1 = \frac{\sum (Y_i - \bar{Y})(X_i - \bar{X})}{\sum (X_i - \bar{X})^2} = \frac{Cov(Y, X)}{Var(X)}$$

Also, $$\beta_1 = \frac{\sum (X_i - \bar{X})(Y_i)}{\sum (X_i - \bar{X})^2} \rightarrow SST_X$$

(Sum of Squares Total on X).

$$= \frac{\sum (X_i - \bar{X})(Y_i)}{SST_X}$$