

## Problem 3

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**Problem 3 (1.3):** Classify the states of the Markov chains whose transition probabilities are given by:

(a)  $p_{02} = 1; p_{11} = 1; p_{i,i-1} = p_{i,i+1} = \frac{1}{2}$

At first we will classify the states of the Markov Chain. Since (1) is an absorption state, it is a separate class.

Also because we cannot go from 2 to 0, (0) is a separate class. Hence following are the classes:

$$(0) (1) (2, 3, \dots)$$

Since  $p_{02} = 1$ , (0) is a transient state.

Since  $p_{11} = 1$ , (1) is an absorption state.

For any state  $i \in (2, 3, \dots)$ ,  $Q = (\frac{q}{p})^{i-1} = 1$  (**from 1.118**)

According to the definition on **pg.15** we get  $L_{ii} < 1$ , and therefore  $i$  is a Transient State.

(b)  $p_{00} = \frac{1}{2}; p_{01} = \frac{1}{2}; p_{i,i-1} = p_{i,i+1} = \frac{1}{2}$

Since we can move from any state  $i$  to any other state  $j$ , all the states are in the same class.

WLOG:  $i < j$ .  $p_{ij}^{(j-i)} = (\frac{1}{2})^{(j-i)} \rightarrow p_{ij} > 0$ . Therefore, there is only one class of states:

$$(0, 1, 2, \dots)$$

Now for any given state  $i \in (0, 1, 2, \dots)$ ,

$$Q = (\frac{q}{p})^i = 1 \quad (\text{from 1.118})$$

Here  $Q$  represents the probability that the particle will ever (or eventually) reach the state 0. Therefore, 0 is a Recurrent state and since all the states are in the same class, they are all Recurrent.

(c)  $p_{00} = \frac{1}{3}; p_{01} = \frac{2}{3}; p_{i,i-1} = \frac{1}{3}; p_{i,i+1} = \frac{2}{3}$

Since we can move from any state  $i$  to any other state  $j$ , all the states are in the same class.

If:  $i < j$ .  $p_{ij}^{(j-i)} = (\frac{2}{3})^{(j-i)} \rightarrow p_{ij} > 0$ .

If:  $i > j$ .  $p_{ij}^{(i-j)} = (\frac{1}{3})^{(j-i)} \rightarrow p_{ij} > 0$ . Therefore, there is only one class of states:

$$(0, 1, 2, \dots)$$

Now for any given state  $i \in (0, 1, 2, \dots)$ ,

$$Q = (\frac{q}{p})^i = (\frac{1}{2})^i < 1. \quad (\text{from 1.118})$$

Here,  $Q$  has the same definition as in part (b). Therefore, 0 is a Transient state and since all the states are in the same class, they are all Transient.