Econometrics (31/March/2016)

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \underbrace{\frac{\sum (x_i - \bar{x})}{SST_X}}_{k_i} y_i$$

$$\hat{\beta}_1 = k_i y_i \implies \sum k_i = 0$$

$$\cancel{\sum k_i y_i}$$

$$\sum k_i x_i = 1$$

$$\sum k_i^2 = \frac{1}{SST_X}$$

$$E(\hat{\beta}_1) = E\left[\sum k_i y_i\right]$$

$$= E\left[\sum k_i(\beta_0 + \beta_1 x + u)\right]$$

$$= \beta_1$$

$$E[\hat{\beta}_0] = E[\bar{y} - \hat{\beta}_1 \bar{x}]$$

$$= E[\beta_0 + \beta_1 \bar{x} + u - \hat{\beta}_1 \bar{x}]$$

$$= \beta_0 \qquad (?)$$

$$Var(\hat{\beta}_1) = Var(\sum k_i y_i) \overset{(?)}{=} \sum k_i^2 Var(y_i)$$

$$= \frac{\sigma^2}{SST_x}$$

$$Var(\hat{\beta}_0) = \sigma^2 \left[\frac{1}{n} + \frac{\bar{x}^2}{SST_x} \right]$$

Gauss-Markov Theorem

Under A1 – A6 ; The OLS estimators ($\hat{\beta}_0, \hat{\beta}_1$) are BLUE.

$$y = 20 + 0.1x + u$$

$$Var(u|x) = \sigma^2 = 1$$

~~$Var(\beta_1) =$~~ $\beta_1 = \frac{y_5 - y_1}{400}$

$$\Rightarrow \quad Var(\beta_1) = \left(\frac{1}{400}\right)^2 \left[Var(y_5) + Var(y_1) \right]$$

$$= \frac{\sigma^2 + \sigma^2}{(400)^2}$$

$$= \frac{\sigma^2}{80{,}000}$$

Pros/Cons

⊕ Simple Estimator

⊕ Simple to Interpret

⊖ Strong assumption (A3)

⊖ ∃ non-linear and more efficient estimators

⊖ OLS is sensitive to outliers.

$y = \beta_0 + \beta_1 x + u \Rightarrow$ Level-Level model

$\log y = \beta_0 + \beta_1 x + u \longrightarrow$ log-level model

$y = \beta_0 + \beta_1 (\log x) + u \longrightarrow$ Level-log "

$\log y = \beta_0 + \beta_1 [\log x] + u \longrightarrow$ log-log "

$$\log y = \beta_1 (\log x_1) + \beta_2 (\log x_2)$$

$$\Longrightarrow \quad \underbrace{\Delta y}_{\text{Unit}(\$)} = \beta_1 \underbrace{(\Delta x)}_{\text{Unit(year)}}$$

log-level model $\Longrightarrow \quad \Delta(\log y) = \beta_1 (\Delta x)$

$$\Rightarrow \quad \%(\Delta y) = (100 \cdot \beta_1)(\Delta x)$$

Level-log model $\Longrightarrow \quad \Delta y = \beta_1 (\Delta \log x)$

If we double our education, the wage will rise by β_1

$$\Rightarrow \quad \Delta y = \frac{\beta_1}{100} (\% \Delta x)$$

log-log-model $\Rightarrow$ $\Delta(\log y) = \beta_1 (\Delta \log x)$

$$\beta_1 = \frac{\Delta y / y}{\Delta x / x}$$

(Elasticity)

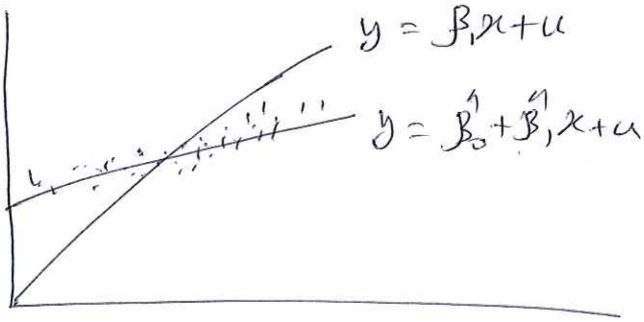
Level-Level $\quad \dfrac{\Delta y}{\Delta x}$

log-Level $\quad \dfrac{\Delta y}{\Delta x} \cdot \dfrac{1}{y}$

Level-log $\quad \dfrac{\Delta y}{\Delta x} \cdot x$

Log-Log $\quad \dfrac{\Delta y}{\Delta x} \cdot \dfrac{x}{y}$

Without Intercept



$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x}) y_i}{\sum (x_i - \bar{x})^2}$$

$$\tilde{\beta}_1 = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$E[\tilde{\beta}_1] = E\left[\frac{\Sigma x_i}{\Sigma x_i^2}(\beta_0 + \beta_1 x_i + u)\right]$$

$$= \beta_0 \frac{\Sigma x_i}{\Sigma x_i^2} + \beta_1 + E\left(\frac{\Sigma x_i u_i}{\Sigma x_i^2}\right)$$

$$= \beta_0 \frac{n\bar{x}}{\Sigma x_i^2} + \beta_1$$

$$\Longrightarrow E[\tilde{\beta}_1] = \beta_1 + \underbrace{\beta_0 \frac{n\bar{x}}{\Sigma x_i^2}}_{\text{Bias}}$$

Hence if $\bar{x} > 0$ and $\beta_0 < 0$

$$\Longrightarrow \text{Bias} < 0$$