

## Problem 6

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**Problem 6 (2.3):** If  $F(s,t)$  is the generating function associated with the random variable  $X(t)$ , show that  $F(1/s,t)$  is the generating function associated with  $-X(t)$ .

We know that:

$$F(s,t) = \sum P_x(t)s^x \quad (1)$$

We introduce a new variable  $Y(t)$  such that  $Y(t) = -X(t)$ . We write the generating function of  $Y(t)$  derived from **(16)**:

$$M(s,t) = \sum P_y(t)s^y \quad (2)$$

By replacing  $y$  by  $-x$  in **(17)** we get:

$$M(s,t) = \sum P_x(t)s^{-x} \quad (3)$$

We solve the problem by stating that:

$$M(s,t) = \sum P_x(t)(1/s)^x = F(1/s,t) \quad (4)$$

Thus we showed that the generating function associated with  $Y(t) = -X(t)$  is the same as  $F(1/s,t)$ .