Problem 8

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Problem 8 (1.8): A Markov Chain with a Transition Matrix P is said to be periodic with period ω if $P^{a+\omega} = P^a$ and ω is the smallest positive integer with this property. Determine the limit matrix Π for periodic chains.

We are given that for a periodic matrix of period ω we have $P^{a+\omega} = P^a$.
We define the Limit Matrix as follows:

$$\Pi = lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} P^i$$

(from Theorem 1.9 on page 36).
At first we will solve the problem for a particular $n \in Z^+$ and then to take the limit matrix we will pass to the limit as $n \to \infty$. We get the following expression for n:

$$\Pi_n = \frac{1}{n} \sum_{i=1}^{n} P^i$$

$$\Pi_n = \frac{1}{n}([\frac{n}{\omega}](P^1 + P^2 + P^3 + ... + P^\omega) + P^1 + P^2 + P^3 + ...P^{n-[\frac{n}{\omega}]\omega})$$

Where $[X]$ is the largest integer greater than or equal to X.

$$\Pi_n = \frac{[\frac{n}{\omega}]}{n}(\sum_{i=1}^{\omega} P^i) + \frac{1}{n}(P^1 + P^2 + P^3 + ... + P^{n-[\frac{n}{\omega}]\omega})$$

We see that as $n \to \infty$

$$\Pi = \frac{1}{\omega}(\sum_{i=1}^{\omega} P^i)$$