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Chap 3: Reflection & Sources

3.3 Diffusion with a Source

$$V_t - kV_{xx} = 0 \quad \rightarrow \text{No source}$$

Consider
$$\left. \begin{aligned} V_t - kV_{xx} &= f(x,t) \\ V(x,0) &= \phi(x) \end{aligned} \right\} (1)$$

Solution of (1)

$$v(x,t) = \int_{-\infty}^{\infty} S(x-y,t) \phi(y) dy + \int_0^t \int_{-\infty}^{\infty} S(x-y,t-s) f(y,s) dy ds$$

U is vector
 A is matrix
 \underline{I} is I.C.

$$\frac{dU}{dt} + AV = F \quad \Rightarrow \quad U(t) = e^{-tA} \underline{I} + \int_0^t e^{-(t-s)A} F(s) ds$$